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DYNAMICAL DETERMINATION OF OHMIC STATES OF A CYLINDRICAL PINCH

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1. INTRODUCTION

One of the fundamental problems in understanding the observed behavior of the Reversed Field Pinch (RFP) is the mechanism for generation and maintenance of the reversed axial field. While the theory of Taylor [1] predicts a relaxation toward such a state, it does not address the problem of the dynamics of its attainment or regeneration. Experimental observations indicate that the mechanism may not be unique [2], and models based on both turbulence [3] and instability [4] have been proposed. In the former case, it is shown that the mean magnetic field evolves from a given initial state to a final force-free steady state consistent with the Taylor theory. In the latter case, the nonlinear interaction of kink modes with different helicity leads to field reversal, which is then sustained by continued unstable activity.

In this paper we address the dual problems of generation and sustainment of the reversed axial field. We show that, if a cylindrical plasma is initially in an axisymmetric state with a sufficient degree of paramagnetism, field reversal can be attained by mode activity of a single helicity. The initial paramagnetism may be due to the method of pinch formation, as in fast experiments [5], or to a gradual altering of the pitch profile resulting from a succession of instabilities [6]. Furthermore, if the total current is kept constant and energy loss and resistivity profiles are included in an ad hoc manner, we find that the final steady state of the helical instability can be maintained for long times against resistive diffusion without the need for further unstable activity. These states, which possess zero order flow and possibly reversed axial field, represent steady equilibria which simultaneously satisfy force balance and Ohm's law, and are termed Ohmic states [7].

2. METHOD OF SOLUTION

We approach the problem of the existence and accessability of Ohmic states by solving an initial value problem in which an initially unstable, axisymmetric cylindrical Z-pinch equilibrium is perturbed with an eigenmode of an $m=1$ resistive kink instability obtained from a linear resistive MHD code. The nonlinear evolution and saturated final state of this configuration is then determined with a nonlinear, two-dimensional MHD code which solves the full set of helically symmetric resistive MHD equations [8]. Specifically, we choose

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$$J_z = \frac{4I}{a^2}\left(1 - \frac{r^2}{a^2}\right) \quad , \quad B_\theta = \frac{2Ir}{a^2}\left(1 - \frac{1}{2}\frac{r^2}{a^2}\right)$$

where r=a is the radius of a conducting wall, and the boundary conditions are such that the total current I remains constant throughout the calculation. Since there is initially no azimuthal current, the axial field $B_Z(r)$ can be specified arbitrarily. Specific choices are discussed in the next section. More peaked or flattened initial current profiles could be used, but here we consider only the parabolic model given above.

In order to approximate the cool outer regions of the pinch, we employ a scalar resistivity model which is peaked near the conducting wall, i.e., $\eta(r)=1/[\sigma_0+(1-r^2/a^2)^2]$, where σ_0 is a constant which limits the value of resistivity at the wall, and is usualy taken to be 10^{-2}. A more complete calculation would allow for a self-consistent resistivity, or perhaps even $\eta=\eta(\psi)$, where ψ is the helical flux. However, the above model has the gross property of maintaining a peaked current profile throughout the evolution of the pinch, while avoiding the nonlinear computational problems associated with a self-consistent calculation.

Since the total axial current remains constant, a steady state will not be reached unless the resulting Ohmic heating is balanced by an energy loss term, which physicaly may be due to radiation, diffusion to the limiter, etc. For these calculations we choose a loss rate profile $q(r)=\alpha(r/a)^2\, p/(\gamma-1)$, which again serves to cool the outer regions of the plasma.

3. RESULTS

For a given value of total current, we allow the instability to proceed to its final saturated helical state where we examine the average value of axial field at the wall, defined be

$$\langle B_{ZW}\rangle = \frac{1}{2\pi}\int_0^{2\pi} B_z(a,\theta)d\theta$$

A plot of $\langle B_{ZW}\rangle_f$, the average value of $B_Z(a)$ in the final Ohmmic state, versus $B_{\theta_W}=I/a$, the value of azimuthal field at the wall, is thus equivalent to the familiar F-θ diagram.

Particular results require that the initial axial field profile $B_Z(r)$ be specified. The choice $B_Z(r)$=const.=1 corresponds to initial conditions which resemble a tokamak, and for a given total current can be made unstable by the proper choice of axial wave number. In figure 1 we plot $\langle B_{ZW}\rangle_f$ versus B_{θ_W} for this case. Note that, while quite paramagnetic final states are reached, field reversal is not achieved, with $\langle B_{ZW}\rangle_f \simeq .35$ for

$B_{\theta_W} \geq 4$. Helical mode activity does not provide sufficient axial flux amplification in the core to reverse the axial field for this configuration. The helical flux topology and flow field for the Ohmic state corresponding to $B_{\theta_W}=1.5$ are shown in figures 2a,b. Note the presence of the magnetic island which persists in spite of resistive diffusion. In figure 4 we display $\langle B_{zW} \rangle$ as a function of time for this case, demonstrating the relaxation to a final Ohmic state.

There is experimental evidence that the initial state of the pinch may be highly paramagnetic [5]. This is certainly the case for fast pinches, and may also result in slow pinches from gradual modification of the pitch profile due to a series of instabilities such as that described above [6]. We thus consider an initial axial field profile of the form

$$B_z = 1 - (1 - B_{zw_o}) \frac{2r^2}{a^2} (1 - \frac{1}{2} \frac{r^2}{a^2})$$

where B_{zw_o} is the value of $B_z(a)$ in the initial state. When $B_{zw_o}=.5$, field reversed Ohmic states are still not achieved. When $B_{zw_o}=.02$, transient reversal is obtained for $B_{\theta_W}=1.5$, and a fully reversed Ohmic equilibrium is achieved when $B_{\theta_W}=2$. This is demonstrated in figure 5, where we plot $\langle B_{zW} \rangle$ versus time. The resulting helical flux surfaces are shown in figure 6. Note that, while the state is helically deformed, no magnetic island is present.

4. DISCUSSION

We have demonstrated qualitatively that, in addition to turbulence and instability, there is a further candidate for the maintenance of the reversed field state, i.e., an Ohmic equilibrium which is maintained against resistive diffusion by the action of zero-order flow. Additionally, such states are accessible as the saturated states of helical instabilities. However, since in our model the resistivity and loss profiles are somewhat arbitrary and not self-consistent, quantitative results are elusive. For example, when proper normalization is taken into account, we find field reversal to occur for values of $\theta=B_{\theta_W}/B_{z_{ave}}$ in the range $4.4<\theta<5.8$, which is considerably larger than predicted by Taylor's theory [1]. This may also be due to the scalar character of the resistivity. In any case, further refinement of the model may be necessary.

5. ACKNOWLEDGEMENT

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FIGURE CAPTIONS

Fig. 1 $\langle B_{zw}\rangle_f$, the average value of the axial field at the wall, vs. B_{θ_w} for the case $B_{zw_o} = 1$.

Fig. 2 Helical flux surfaces in the final Ohmic corresponding to $B_{\theta_w}=1.5$, $B_{zw_o}=1$.

Fig. 3 Final flow field corresponding to fig. 2.

Fig. 4 $\langle B_{zw}\rangle$ vs. time for the case $B_{\theta_w}=1.5$, $B_{zw_o}=1$.

Fig. 5 $\langle B_{zw}\rangle$ vs. time for the case $B_{\theta_w}=2.0$, $B_{zw_o}=.02$. Note that a field-reversed Ohmic state has been attained.

Fig. 6 Helical flux contours in the Ohmic state $B_{\theta_w}=2.0$, $B_{zw_o}=.02$. Note the absence of a magnetic island.

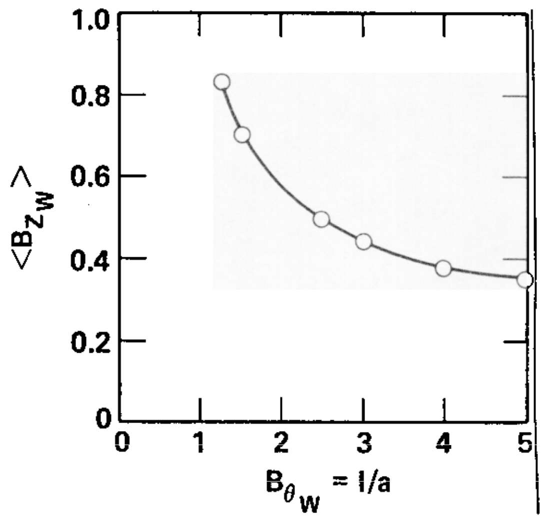

Fig. 1

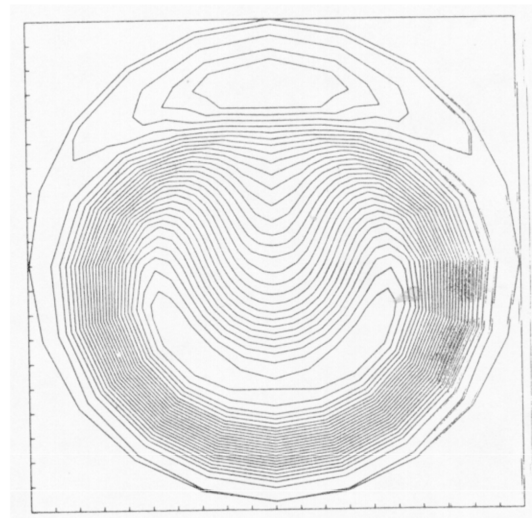

Fig. 2

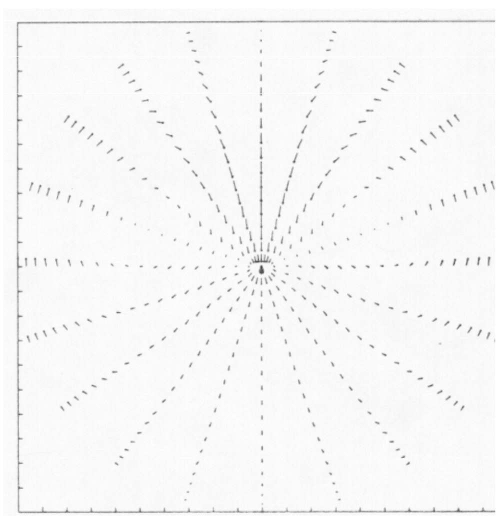

Fig. 3

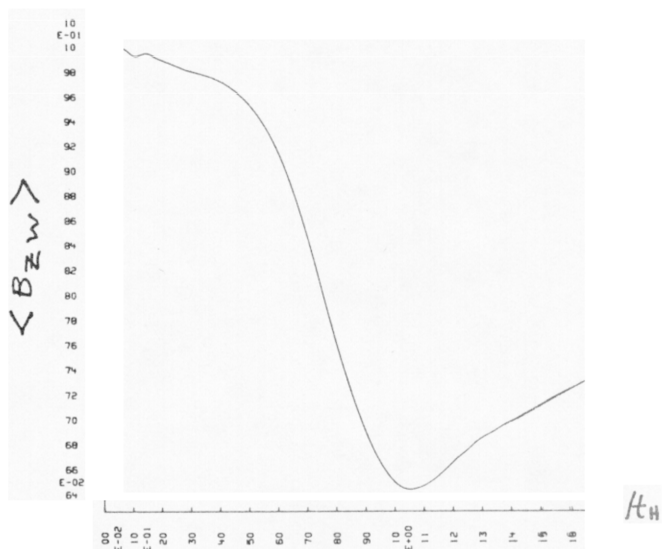

Fig. 4

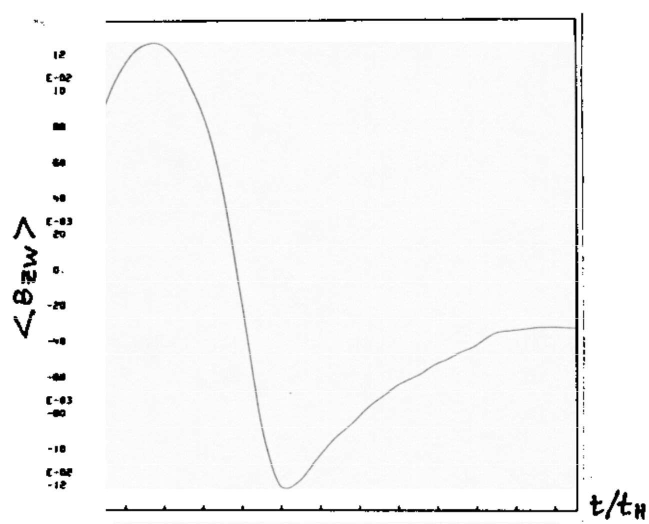

Fig. 5

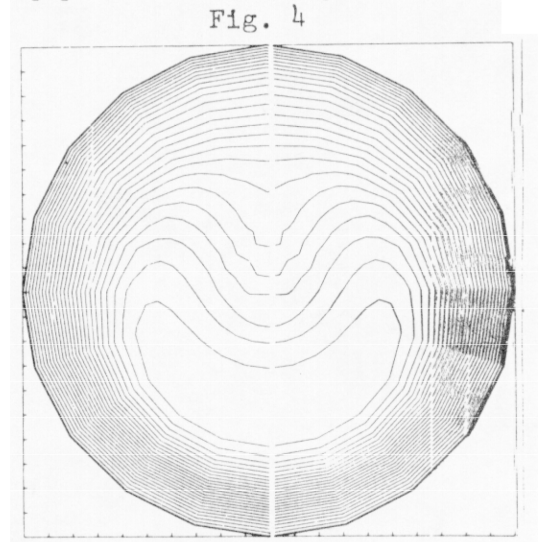

Fig. 6